**Profit Max w/ Math**
Econ 201/Haworth

Assume a typical monopoly can be described by the following equations (note that because this firm is a monopoly, we can use lower case q or upper case Q to represent the firm's output):

| | |
|---|---|
| $P = 180 - q$ | (Demand) |
| $MR = 180 - 2q$ | (Marginal Revenue) |
| $TC = 0.5q^3 - 4q^2 + 5q + 100$ | (Total Cost) |
| $MC = 1.5q^2 - 8q + 5$ | (Marginal Cost) |

If we convert TC into AC, we have: $AC = 0.5q^2 - 4q + 5 + \frac{100}{q}$

Let's solve for q*, the profit maximizing level of output.

Set MR = MC, and solve for q

$MR = MC$
$180 - 2q = 1.5q^2 - 8q + 5$
$1.5q^2 - 6q - 175 = 0$

Using the quadratic equation, we get: q* = 12.99 and q* = -8.99

**That was too much work, so we'll try something else!**

Now, let's assume that this firm has a new TC and MC equation, where TC = 60q and MC = 60, instead of $TC = 0.5q^3 - 4q^2 + 5q + 100$ and $MC = 1.5q^2 - 8q + 5$)

That gives us the following new set of equations:

| | |
|---|---|
| $P = 180 - q$ | (Demand) |
| $MR = 180 - 2q$ | (Marginal Revenue) |
| $AC = 60$ | (Average Cost) |
| $MC = 60$ | (Marginal Cost) |

Now, let's set MR = MC, and solve for the new q

$MR = MC$
$180 - 2q = 60$
$q^* = 60$

Plug q* = 60 into the demand curve equation:

$P = 180 – q$
$P = 180 – (60)$
$P = 120$
$P^* = \$120$

We now calculate profit as $\pi = (P - AC)\, q$

$\pi = (P - AC)\, q$

$\pi = (120 - 60)\, 60$

$\pi = \$3600$

Assuming that AC and MC are constant obviously makes this problem much easier, but how realistic is this assumption of constant AC and MC?

First, we want to note that every firm can potential have a different TC, which gives us different AC and MC curves across firms. When MC is constant, that is saying we have a firm who pays the same amount in cost for every extra unit.

Suppose we have a Louisville-based airline with regular flights going to Chicago. Let's assume that the output of this firm is the service provided to individual passengers of getting them from Louisville to Chicago. To provide this service, the firm would need gasoline, flight attendants, pilots, etc. Would these costs vary with output though (i.e. # of passengers)? No, the airline is likely going to spend the same overall amount of money on flight attendants, pilots, gasoline, etc, whether the plane has 1 passenger or 100 passengers.

However, one cost is very likely going to vary with output (# passengers), and that's food and drinks. If we assume that the airline provides \$10 in food and drinks to each passenger on this flight, then how would that cost vary with output (# passengers)? The overall cost would increase by \$10 with every passenger on the flight. In other words, the airline would have a constant \$10 marginal cost associated with each flight. Every additional unit of output (passenger) will increase the airline's cost by \$10. This is an example of constant marginal cost.

Do all firms have constant marginal cost? No, of course not, but we do find examples of constant marginal cost, which tells us that this assumption is not completely unrealistic.

***Why is this handout important?*** *This handout illustrates how mathematical examples in this class must proceed when it comes to doing a profit max problem with monopoly. The moral here is that any profit max question in the monopoly section must involve a monopoly firm with constant marginal cost. I.e., we will have a marginal cost equation that's equal to a constant value (e.g. MC = 60) and a marginal cost curve that would be a flat (horizontal) line on a graph.*